

平方根の加減

例題 次の計算をせよ。(有理化して計算)

(1) $\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{3}$

(2) $\sqrt{96} - \sqrt{\frac{2}{3}}$

(3) $\frac{\sqrt{5}}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{5}}$

答

答

答

練習A 次の計算をせよ。

(1) $\frac{1}{\sqrt{2}} - \frac{3\sqrt{2}}{2}$

(3) $\sqrt{18} - \frac{1}{\sqrt{2}}$

答

答

(2) $\frac{2}{\sqrt{2}} - \frac{6}{\sqrt{3}}$

(4) $\sqrt{\frac{3}{5}} + \frac{2\sqrt{5}}{\sqrt{3}}$

答

答

練習B 次の計算をせよ。

(1) $\sqrt{14} - \frac{\sqrt{2}}{\sqrt{7}}$

答

(2) $\frac{4}{\sqrt{2}} + \frac{6}{\sqrt{3}}$

答

(3) $\frac{5}{2\sqrt{3}} - \frac{3}{\sqrt{27}}$

答

(4) $5\sqrt{2} - \frac{4}{\sqrt{2}}$

答

(5) $\frac{\sqrt{10}}{\sqrt{2}} + \frac{3}{\sqrt{5}}$

答

(6) $\sqrt{27} - \frac{6}{\sqrt{3}} - 3\sqrt{12}$

答

練習C 次の計算をせよ。

(1) $\frac{4\sqrt{10}}{5} + \frac{\sqrt{2}}{\sqrt{5}}$

答

(2) $2\sqrt{\frac{8}{9}} - \sqrt{72}$

答

(3) $6\sqrt{\frac{5}{3}} + \sqrt{60}$

答

(4) $\sqrt{\frac{9}{7}} - \sqrt{\frac{7}{9}}$

答

(5) $-\frac{2}{\sqrt{3}} + \frac{5\sqrt{2}}{\sqrt{6}}$

答

(6) $3\sqrt{2} + \frac{\sqrt{18}}{3} - \frac{4\sqrt{5}}{\sqrt{10}}$

答

練習D 次の計算をせよ。

(1) $\frac{\sqrt{20}}{2} - \sqrt{\frac{5}{4}} + 2\sqrt{5}$

答

(2) $\frac{\sqrt{75}}{2} + \sqrt{12} - 3\sqrt{27} - \sqrt{\frac{3}{4}}$

答

(3) $\sqrt{\frac{3}{2}} + \sqrt{24} + \left(-\frac{\sqrt{54}}{2}\right)$

答

(4) $\sqrt{0.63} - \sqrt{1.75} + \sqrt{0.28}$

答

(5) $4\sqrt{5} - \sqrt{20} + \frac{7}{2\sqrt{5}} - \sqrt{45}$

答

(6) $\frac{4\sqrt{45} + 2\sqrt{20} - \sqrt{180}}{\sqrt{10}}$

答

平方根の加減

例題 次の計算をせよ。(有理化して計算)

(1) $\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{3}$

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$$\begin{aligned} \frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{3} &= \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} - \frac{\sqrt{3}}{3} \\ &= \frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{3} \\ &= \frac{\sqrt{3}}{3} \end{aligned}$$

答 $\frac{\sqrt{3}}{3}$

(2) $\sqrt{96} - \sqrt{\frac{2}{3}}$

★

$$\begin{aligned} \sqrt{96} - \sqrt{\frac{2}{3}} &= 4\sqrt{6} - \frac{\sqrt{2}}{\sqrt{3}} \\ &= 4\sqrt{6} - \frac{\sqrt{2} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ &= 4\sqrt{6} - \frac{\sqrt{6}}{3} \\ &= \frac{12\sqrt{6}}{3} - \frac{\sqrt{6}}{3} \\ &= \frac{11\sqrt{6}}{3} \end{aligned}$$

答 $\frac{11\sqrt{6}}{3}$

(3) $\frac{\sqrt{5}}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{5}}$

★

$$\begin{aligned} \frac{\sqrt{5}}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{5}} &= \frac{\sqrt{5} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} - \frac{\sqrt{3} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} \\ &= \frac{\sqrt{15}}{3} - \frac{\sqrt{15}}{5} \\ &= \frac{5\sqrt{15}}{15} - \frac{3\sqrt{15}}{15} \\ &= \frac{2\sqrt{15}}{15} \end{aligned}$$

答 $\frac{2\sqrt{15}}{15}$

練習A 次の計算をせよ。

(1) $\frac{1}{\sqrt{2}} - \frac{3\sqrt{2}}{2}$

★

$$\begin{aligned} \frac{1}{\sqrt{2}} - \frac{3\sqrt{2}}{2} &= \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} - \frac{3\sqrt{2}}{2} \\ &= \frac{\sqrt{2}}{2} - \frac{3\sqrt{2}}{2} \\ &= -\frac{2\sqrt{2}}{2} \\ &= -\sqrt{2} \end{aligned}$$

答 $-\sqrt{2}$

(2) $\frac{2}{\sqrt{2}} - \frac{6}{\sqrt{3}}$

★

$$\begin{aligned} \frac{2}{\sqrt{2}} - \frac{6}{\sqrt{3}} &= \frac{2 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} - \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ &= \frac{2\sqrt{2}}{2} - \frac{6\sqrt{3}}{3} \\ &= \sqrt{2} - 2\sqrt{3} \end{aligned}$$

答 $\sqrt{2} - 2\sqrt{3}$

(3) $\sqrt{18} - \frac{1}{\sqrt{2}}$

★

$$\begin{aligned} \sqrt{18} - \frac{1}{\sqrt{2}} &= 3\sqrt{2} - \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\ &= 3\sqrt{2} - \frac{\sqrt{2}}{2} \\ &= \frac{6\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \\ &= \frac{5\sqrt{2}}{2} \end{aligned}$$

答 $\frac{5\sqrt{2}}{2}$

(4) $\sqrt{\frac{3}{5}} + \frac{2\sqrt{5}}{\sqrt{3}}$

★

$$\begin{aligned} \sqrt{\frac{3}{5}} + \frac{2\sqrt{5}}{\sqrt{3}} &= \frac{\sqrt{3}}{\sqrt{5}} + \frac{2\sqrt{5}}{\sqrt{3}} \\ &= \frac{\sqrt{3} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} + \frac{2\sqrt{5} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ &= \frac{\sqrt{15}}{5} + \frac{2\sqrt{15}}{3} \\ &= \frac{3\sqrt{15}}{15} + \frac{10\sqrt{15}}{15} \\ &= \frac{13\sqrt{15}}{15} \end{aligned}$$

答 $\frac{13\sqrt{15}}{15}$

練習B 次の計算をせよ。

$$(1) \sqrt{14} - \frac{\sqrt{2}}{\sqrt{7}}$$

★

$$\begin{aligned} \sqrt{14} - \frac{\sqrt{2}}{\sqrt{7}} &= \sqrt{14} - \frac{\sqrt{2} \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} \\ &= \sqrt{14} - \frac{\sqrt{14}}{7} \\ &= \frac{7\sqrt{14}}{7} - \frac{\sqrt{14}}{7} \\ &= \frac{6\sqrt{14}}{7} \end{aligned}$$

答

$$\frac{6\sqrt{14}}{7}$$

$$(2) \frac{4}{\sqrt{2}} + \frac{6}{\sqrt{3}}$$

★

$$\begin{aligned} \frac{4}{\sqrt{2}} + \frac{6}{\sqrt{3}} &= \frac{4 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ &= \frac{4\sqrt{2}}{2} + \frac{6\sqrt{3}}{3} \\ &= 2\sqrt{2} + 2\sqrt{3} \end{aligned}$$

答

$$2\sqrt{2} + 2\sqrt{3}$$

$$(3) \frac{5}{2\sqrt{3}} - \frac{3}{\sqrt{27}}$$

★

$$\begin{aligned} \frac{5}{2\sqrt{3}} - \frac{3}{\sqrt{27}} &= \frac{5}{2\sqrt{3}} - \frac{3}{3\sqrt{3}} \\ &= \frac{5 \times \sqrt{3}}{2\sqrt{3} \times \sqrt{3}} - \frac{3 \times \sqrt{3}}{3\sqrt{3} \times \sqrt{3}} \\ &= \frac{5\sqrt{3}}{6} - \frac{3\sqrt{3}}{9} \\ &= \frac{15\sqrt{3}}{18} - \frac{6\sqrt{3}}{18} = \frac{9\sqrt{3}}{18} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

答

$$\frac{\sqrt{3}}{2}$$

$$(4) 5\sqrt{2} - \frac{4}{\sqrt{2}}$$

★

$$\begin{aligned} 5\sqrt{2} - \frac{4}{\sqrt{2}} &= 5\sqrt{2} - \frac{4 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\ &= 5\sqrt{2} - \frac{4\sqrt{2}}{2} \\ &= 3\sqrt{2} \end{aligned}$$

答

$$3\sqrt{2}$$

$$(5) \frac{\sqrt{10}}{\sqrt{2}} + \frac{3}{\sqrt{5}}$$

★

$$\begin{aligned} \frac{\sqrt{10}}{\sqrt{2}} + \frac{3}{\sqrt{5}} &= \frac{\sqrt{10} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{3 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} \\ &= \frac{\sqrt{20}}{2} + \frac{3\sqrt{5}}{5} \\ &= \frac{2\sqrt{5}}{2} + \frac{3\sqrt{5}}{5} \\ &= \frac{5\sqrt{5}}{5} + \frac{3\sqrt{5}}{5} \\ &= \frac{8\sqrt{5}}{5} \end{aligned}$$

答

$$\frac{8\sqrt{5}}{5}$$

$$(6) \sqrt{27} - \frac{6}{\sqrt{3}} - 3\sqrt{12}$$

★

$$\begin{aligned} \sqrt{27} - \frac{6}{\sqrt{3}} - 3\sqrt{12} &= 3\sqrt{3} - \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} - 3 \times 2\sqrt{3} \\ &= 3\sqrt{3} - \frac{6\sqrt{3}}{3} - 6\sqrt{3} \\ &= 3\sqrt{3} - 2\sqrt{3} - 6\sqrt{3} \\ &= -5\sqrt{3} \end{aligned}$$

答

$$-5\sqrt{3}$$

練習C 次の計算をせよ。

(1) $\frac{4\sqrt{10}}{5} + \frac{\sqrt{2}}{\sqrt{5}}$

★

$$\begin{aligned} \frac{4\sqrt{10}}{5} + \frac{\sqrt{2}}{\sqrt{5}} &= \frac{4\sqrt{10}}{5} + \frac{\sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} \\ &= \frac{4\sqrt{10}}{5} + \frac{\sqrt{10}}{5} \\ &= \frac{5\sqrt{10}}{5} \\ &= \sqrt{10} \end{aligned}$$

答 $\sqrt{10}$

(2) $2\sqrt{\frac{8}{9}} - \sqrt{72}$

★

$$\begin{aligned} 2\sqrt{\frac{8}{9}} - \sqrt{72} &= \frac{2 \times \sqrt{8}}{\sqrt{9}} - 6\sqrt{2} \\ &= \frac{2 \times 2\sqrt{2}}{3} - 6\sqrt{2} \\ &= \frac{4\sqrt{2}}{3} - 6\sqrt{2} \\ &= \frac{4\sqrt{2}}{3} - \frac{18\sqrt{2}}{3} \\ &= -\frac{14\sqrt{2}}{3} \end{aligned}$$

答 $-\frac{14\sqrt{2}}{3}$

(3) $6\sqrt{\frac{5}{3}} + \sqrt{60}$

★

$$\begin{aligned} 6\sqrt{\frac{5}{3}} + \sqrt{60} &= \frac{6\sqrt{5} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} + 2\sqrt{15} \\ &= \frac{6\sqrt{15}}{3} + 2\sqrt{15} \\ &= 4\sqrt{15} \end{aligned}$$

答 $4\sqrt{15}$

(4) $\sqrt{\frac{9}{7}} - \sqrt{\frac{7}{9}}$

★

$$\begin{aligned} \sqrt{\frac{9}{7}} - \sqrt{\frac{7}{9}} &= \frac{\sqrt{9}}{\sqrt{7}} - \frac{\sqrt{7}}{\sqrt{9}} \\ &= \frac{3}{\sqrt{7}} - \frac{\sqrt{7}}{3} \\ &= \frac{3 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} - \frac{\sqrt{7}}{3} \\ &= \frac{3\sqrt{7}}{7} - \frac{\sqrt{7}}{3} = \frac{9\sqrt{7}}{21} - \frac{7\sqrt{7}}{21} \\ &= \frac{2\sqrt{7}}{21} \end{aligned}$$

答 $\frac{2\sqrt{7}}{21}$

(5) $-\frac{2}{\sqrt{3}} + \frac{5\sqrt{2}}{\sqrt{6}}$

★

$$\begin{aligned} -\frac{2}{\sqrt{3}} + \frac{5\sqrt{2}}{\sqrt{6}} &= -\frac{2}{\sqrt{3}} + \frac{5\sqrt{2}^1}{\sqrt{6}^3} \\ &= -\frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} + \frac{5 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ &= -\frac{2\sqrt{3}}{3} + \frac{5\sqrt{3}}{3} \\ &= \frac{3\sqrt{3}}{3} \\ &= \sqrt{3} \end{aligned}$$

答 $\sqrt{3}$

(6) $3\sqrt{2} + \frac{\sqrt{18}}{3} - \frac{4\sqrt{5}}{\sqrt{10}}$

★

$$\begin{aligned} 3\sqrt{2} + \frac{\sqrt{18}}{3} - \frac{4\sqrt{5}}{\sqrt{10}} &= 3\sqrt{2} + \frac{3\sqrt{2}^1}{3} - \frac{4\sqrt{5}^1}{\sqrt{10}^2} \\ &= 3\sqrt{2} + \sqrt{2} - \frac{4 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\ &= 4\sqrt{2} - \frac{4 \times \sqrt{2}}{2} \\ &= 4\sqrt{2} - 2\sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

答 $2\sqrt{2}$

練習D 次の計算をせよ。

(1) $\frac{\sqrt{20}}{2} - \sqrt{\frac{5}{4}} + 2\sqrt{5}$

★

$$\begin{aligned} \frac{\sqrt{20}}{2} - \sqrt{\frac{5}{4}} + 2\sqrt{5} &= \frac{2\sqrt{5}}{2} - \frac{\sqrt{5}}{\sqrt{4}} + 2\sqrt{5} \\ &= \sqrt{5} - \frac{\sqrt{5}}{2} + 2\sqrt{5} \\ &= 3\sqrt{5} - \frac{\sqrt{5}}{2} \\ &= \frac{6\sqrt{5}}{2} - \frac{\sqrt{5}}{2} \\ &= \frac{5\sqrt{5}}{2} \end{aligned}$$

答 $\frac{5\sqrt{5}}{2}$

(2) $\frac{\sqrt{75}}{2} + \sqrt{12} - 3\sqrt{27} - \sqrt{\frac{3}{4}}$

★

$$\begin{aligned} \frac{\sqrt{75}}{2} + \sqrt{12} - 3\sqrt{27} - \sqrt{\frac{3}{4}} &= \frac{5\sqrt{3}}{2} + 2\sqrt{3} - 3 \times 3\sqrt{3} - \frac{\sqrt{3}}{\sqrt{4}} \\ &= \frac{5\sqrt{3}}{2} + 2\sqrt{3} - 9\sqrt{3} - \frac{\sqrt{3}}{2} \\ &= \frac{2\sqrt{3}}{2} - 7\sqrt{3} \\ &= 2\sqrt{3} - 7\sqrt{3} \\ &= -5\sqrt{3} \end{aligned}$$

答 $-5\sqrt{3}$

(3) $\sqrt{\frac{3}{2}} + \sqrt{24} + \left(-\frac{\sqrt{54}}{2}\right)$

★

$$\begin{aligned} \sqrt{\frac{3}{2}} + \sqrt{24} + \left(-\frac{\sqrt{54}}{2}\right) &= \frac{\sqrt{3}}{\sqrt{2}} + 2\sqrt{6} - \frac{3\sqrt{6}}{2} \\ &= \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + 2\sqrt{6} - \frac{3\sqrt{6}}{2} = \frac{\sqrt{6}}{2} + 2\sqrt{6} - \frac{3\sqrt{6}}{2} \\ &= \frac{2\sqrt{6}}{2} + 2\sqrt{6} \\ &= -\sqrt{6} + 2\sqrt{6} \\ &= \sqrt{6} \end{aligned}$$

答 $\sqrt{6}$

(4) $\sqrt{0.63} - \sqrt{175} + \sqrt{0.28}$

★

$$\begin{aligned} \sqrt{0.63} - \sqrt{175} + \sqrt{0.28} &= \sqrt{\frac{63}{100}} - \sqrt{\frac{175}{100}} + \sqrt{\frac{28}{100}} \\ &= \frac{\sqrt{63}}{\sqrt{100}} - \frac{\sqrt{175}}{\sqrt{100}} + \frac{\sqrt{28}}{\sqrt{100}} \\ &= \frac{3\sqrt{7}}{10} - \frac{5\sqrt{7}}{10} + \frac{2\sqrt{7}}{10} \\ &= 0 \end{aligned}$$

答 0

(5) $4\sqrt{5} - \sqrt{20} + \frac{7}{2\sqrt{5}} - \sqrt{45}$

★

$$\begin{aligned} 4\sqrt{5} - \sqrt{20} + \frac{7}{2\sqrt{5}} - \sqrt{45} &= 4\sqrt{5} - 2\sqrt{5} + \frac{7 \times \sqrt{5}}{2\sqrt{5} \times \sqrt{5}} - 3\sqrt{5} \\ &= -\sqrt{5} + \frac{7\sqrt{5}}{10} \\ &= -\frac{10\sqrt{5}}{10} + \frac{7\sqrt{5}}{10} \\ &= -\frac{3\sqrt{5}}{10} \end{aligned}$$

答 $-\frac{3\sqrt{5}}{10}$

(6) $\frac{4\sqrt{45} + 2\sqrt{20} - \sqrt{180}}{\sqrt{10}}$

★

$$\begin{aligned} \frac{4\sqrt{45} + 2\sqrt{20} - \sqrt{180}}{\sqrt{10}} &= \frac{4 \times 3\sqrt{5} + 2 \times 2\sqrt{5} - 6\sqrt{5}}{\sqrt{10}} \\ &= \frac{12\sqrt{5} + 4\sqrt{5} - 6\sqrt{5}}{\sqrt{10}} \\ &= \frac{10\sqrt{5}}{\sqrt{10 \times 2}} = \frac{10 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\ &= \frac{10\sqrt{2}}{2} \\ &= 5\sqrt{2} \end{aligned}$$

答 $5\sqrt{2}$